An Open Question with NonLocal Boxes

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Objective

Find a criteria explaining why *post-quantum correlations* are unlikely to exist in Nature.

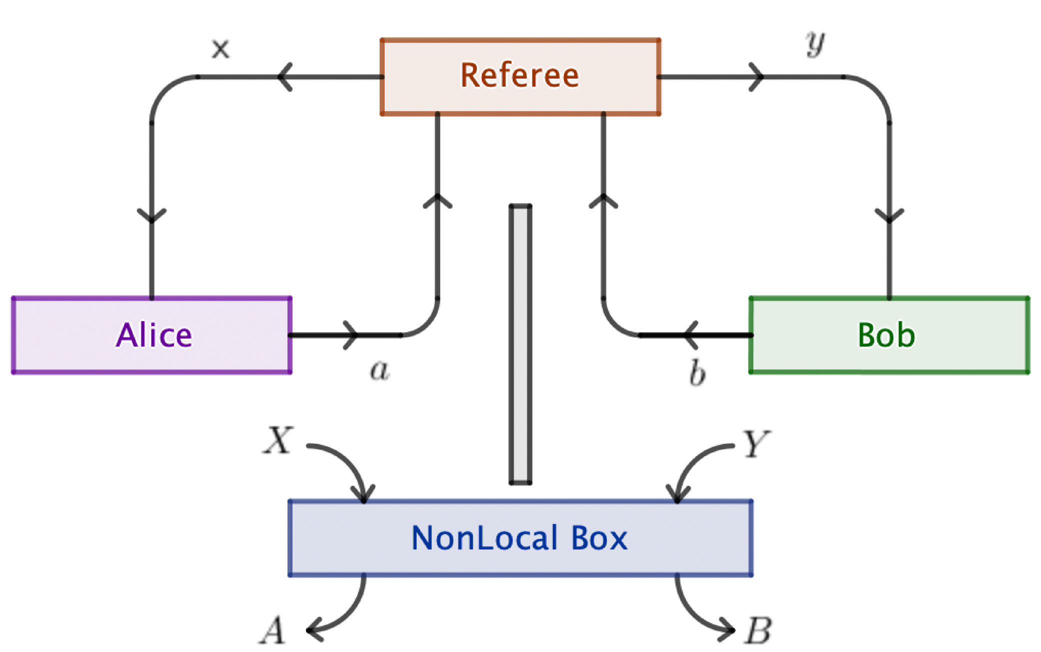
Introduction

Context. At CHSH game, quantum strategies are limited by the so-called Tsirelson's bound, meaning that they can win at best with probability $\approx 85\%$. Nevertheless, post-quantum strategies, formalized by *nonlocal boxes*, can win up to 100% of probability.

Conjecture. It is conjectured that *trivial communication complexity* is a characterization of those post-quantum boxes, explaining why they seem to be implausible in Nature.

CHSH game

Alice and Bob receive bits $x, y \in \{0,1\}$, and they send bits $a, b \in \{0,1\}$ to the referee.



- **Win at CHSH** *iff* $a \oplus b = x \times y$.
- **Win at CHSH′** *iff* $a \oplus b = (x \oplus 1) \times (y \oplus 1)$.

Depending on the ressources they are allowed to use, Alice and Bob have different strategies to win at CHSH:

- **Classical Strategy.** $\max \mathbf{P}\begin{pmatrix}\text{win}\\ \text{CHSH}\end{pmatrix} = 75\%$.
- **Quantum Strategy.** $\max \mathbf{P}\begin{pmatrix}\text{win}\\ \text{CHSH}\end{pmatrix} = \frac{2+\sqrt{2}}{4} \approx 85\%$.
- **Post-Quantum Strategy.** $\max \mathbf{P}\begin{pmatrix}\text{win}\\ \text{CHSH}\end{pmatrix} = 100\%$.
  $\rightsquigarrow$ Framework: *nonlocal boxes.*

NonLocal Boxes

Def. A *nonlocal box* is the data of $\mathbf{P}(A, B \mid X, Y)$ for any $A, B, X, Y \in \{0,1\}$.

Examples.
- $\mathbf{P}_{\text{PR}}(a, b \mid x, y) := \begin{cases} 1/2 & \text{if } a \oplus b = x \times y, \\ 0 & \text{otherwise.} \end{cases}$
- $\mathbf{P}_{\text{SR}}(a, b \mid x, y) := \begin{cases} 1/2 & \text{if } a = b, \\ 0 & \text{otherwise.} \end{cases}$
- $\mathbf{P}_{\text{PR}'}(a, b \mid x, y) := \begin{cases} 1/2 & \text{if } a \oplus b = (x \oplus 1) \times (y \oplus 1), \\ 0 & \text{otherwise.} \end{cases}$

Communication Complexity

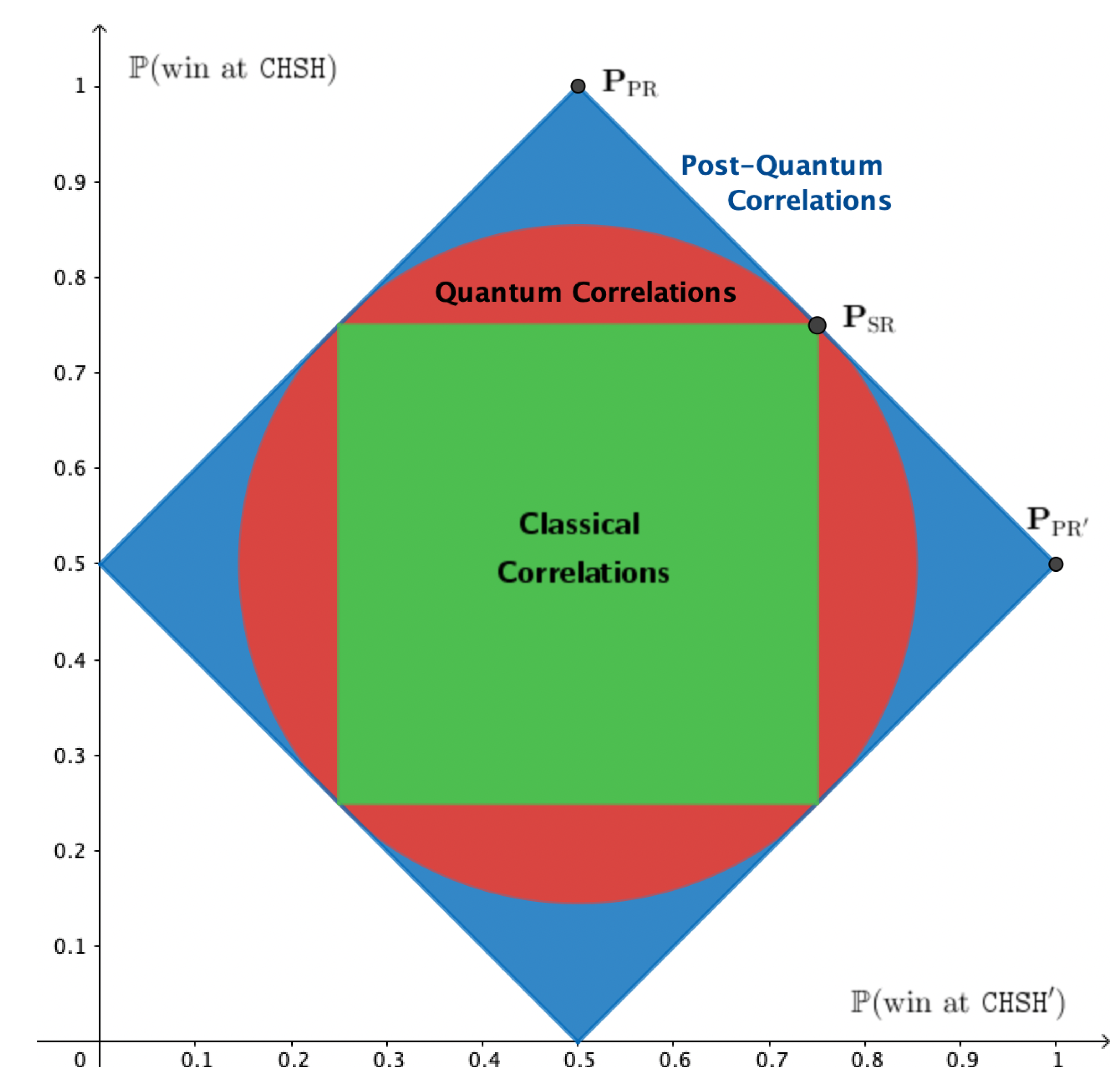
Let $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$. Alice receives x and f, and Bob receives y and f.

Def. The *(probabilistic) communication complexity* of f is the minimal number $\mathbf{CC}_p(f, x, y)$ of bits that Alice and Bob need to exchange so that Alice knows the value $f(x, y)$ with probability $> p$.

Def. Communication complexity is *trivial* if there are $p > 1/2$ and $C > 0$ such that $\mathbf{CC}_p(f, x, y) < C$ for all Boolean function f and all strings $x, y \in \{0,1\}^n$.

Trade-Off between CHSH and CHSH′

For each box $\mathbf{P}$, we compute its probability of winning at CHSH, and the one winning at CHSH′, and it gives the following diagram:



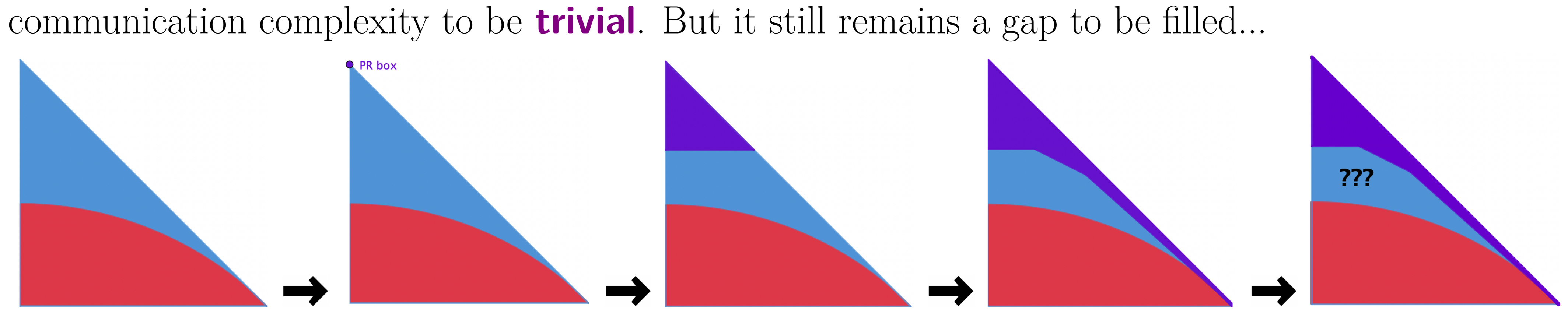
Remark. We find again the values 75%, $\approx 85\%$ and 100% of the three types of strategies mentioned before.

Open Question

Is it possible to distinguish quantum correlations
from post-quantum correlations
using only communication complexity?

Historical Overview

The Question is Partly Answered. Let's *zoom in* at the top-right corner of the diagram. It is known that all **quantum** correlations make communication complexity to be **nontrivial**, whereas some **post-quantum** boxes make communication complexity to be **trivial**. But it still remains a gap to be filled...



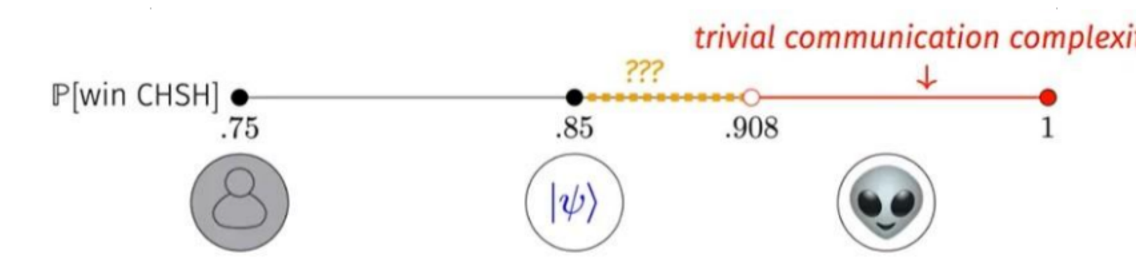
1997: quantum correlations are **non-trivial** [3]. → **2000**: the PR-box is **trivial** [8]. → **2006**: **trivial** zone above ≈ 0.91 [5, 1]. → **2009**: the 'thickened' diagonal is **trivial** [4, 2]. → **Today**: Unknown zone.

Our (coming) Contribution

New. Generalizing the ideas of [2], we are currently introducing an *algebra* on boxes and the notion of the *orbit* of a box, which allows us to thicken a bit the area found in 2009. Tests are in progress ;-)

Conclusion

Today. Until today, there is still a gap to be filled:



and it seems to be a difficult task to fill it [7, 6].

But if indeed all post-quantum boxes were making communication complexity to be trivial, then we would have a clear split between quantum strategies (nontrivial) and post-quantum strategies (trivial).

Consequence. Therefore, the presence of Tsirelson's bound in Quantum Mechanics would simply be understood as a consequence of the "axiom" that communication complexity must be nontrivial for a strategy to exist in Nature!

References

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