# Does the Uncloneable Bit Exist?

Pierre Botteron

(Ottawa, Friday December 8, 2023.)

# Ongoing Work with...

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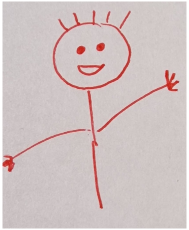

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Clément Pellegrini
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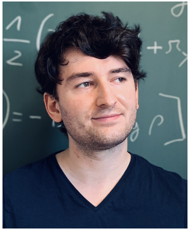

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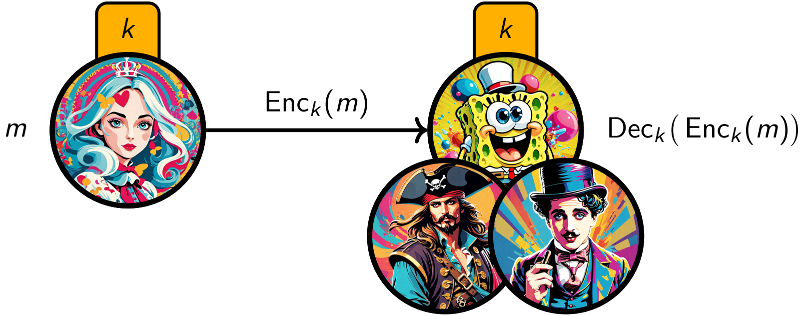
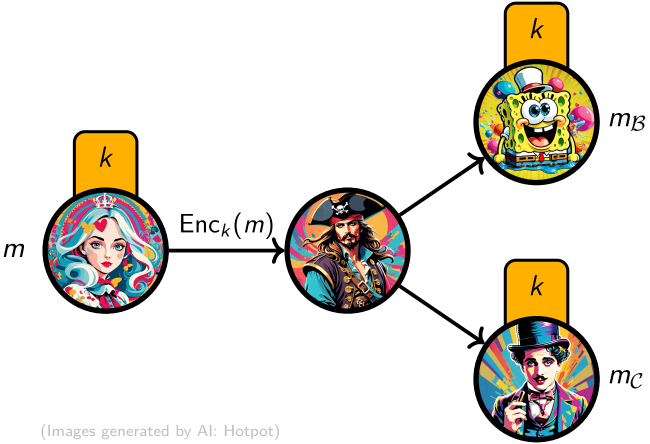



**Correctness:** $\mathsf{Dec}_k(\mathsf{Enc}_k(m)) \stackrel{a.s.}{=} m$.

(Images generated by AI: Hotpot)

# The Cloning Game




(Images generated by AI: Hotpot)

- **Rule:** $\mathcal{P}, \mathcal{B}, \mathcal{C}$ win iff. $m = m_{\mathcal{B}} = m_{\mathcal{C}}$.
- If $\text{Enc}_k(m)$ is classical, then $\mathbb{P}(\mathcal{P}, \mathcal{B}, \mathcal{C} \text{ win}) = 1$. So we are intered in $\text{Enc}_k(m) \in \mathcal{H}$ quantum state.
- If $m \in \{0,1\}^n$ and $\mathcal{P}$ sends a uniformly random message $m_{\mathcal{B}} = m_{\mathcal{C}}$ to $\mathcal{B}, \mathcal{C}$, then $\mathbb{P}(\mathcal{P}, \mathcal{B}, \mathcal{C} \text{ win}) = 1/2^n = 0.5^n$.
- **Open problem:** Find an encryption scheme that is "secure".

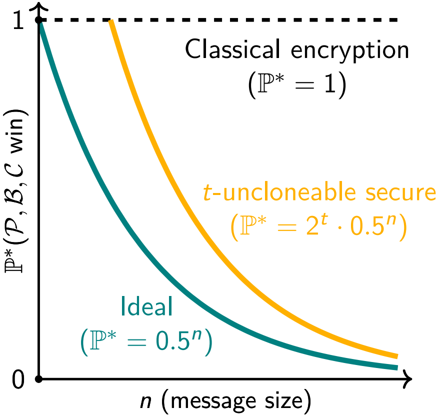
# Uncloneable Security[1]

**Definition.** The encryption scheme $\mathsf{Enc}_k$ is said to be $t(\lambda)$-**uncloneable secure**, with $0 \leq t(\lambda) \leq n$, if the optimal winning probability is "almost" the random one:

$$\mathbb{P}^*(\mathcal{P}, \mathcal{B}, \mathcal{C} \text{ win}) \leq 2^{t(\lambda)} \cdot 0.5^n + \text{negl.}(\lambda)\,,$$

where $\lambda \in \mathbb{N}$ is the security parameter, and $n$ is the size of the message $m$.

**Remarks.**
- $t = 0$ is ideal.
- $t = n$ is trivial.




[1] Broadbent and Lord. *Uncloneable Quantum Encryption via Oracles*. 2020.

— *Part 2* —

# Known Results

# Open Question

- Gottesman[2] introduced a scheme that detects if an adversary could have had information about the plaintext when it was ecnrypted.
- **Open Question.** Is it possible to find an ecryption scheme that would prevent the splitting of a ciphertext?

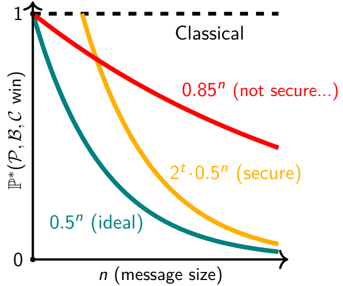
[2] Gottesman. "Uncloneable Encryption". In: *Quantum Info. Comput.* (2003).

# Attempt Without Assumption

**Encryption scheme:** $\mathcal{A}$ encrypts her message $m \in \{0,1\}^n$ in a Wiesner state $|m^k\rangle := H^{k_1}|m_1\rangle \otimes \cdots \otimes H^{k_n}|m_n\rangle$, with a key $k \in \{0,1\}^n$:

$$\mathrm{Enc}_k(m) := |m^k\rangle\langle m^k| \,.$$

**Decryption scheme:** $\mathrm{Dec}_k(\rho) :=$ measurement of $H^k \rho H^k$ in the computational basis.




**Theorem ([Tomamichel – Fehr – Kaniewski – Wehner][3])**

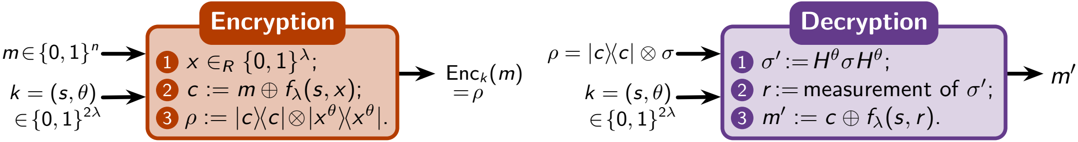
*Using this* $\mathrm{Enc}_k$*, no matter what* $\mathcal{P},\mathcal{B},\mathcal{C}$ *do, their winning probability is bounded by:*

$$\mathbb{P}(\mathcal{P},\mathcal{B},\mathcal{C}\text{ win}) \leq \left(\cos^2(\pi/8)\right)^n \approx 0.854^n \,.$$

[3] Tomamichel et al. “A monogamy-of-entanglement game with applications to device-independent quantum cryptography”. In: *New Journal of Physics* (2013).

# Attempt in the Quantum Random Oracle Model

- **Definition.** "A **quantum-secure pseudorandom function (qPRF)** is a keyed function $f_\lambda : \{0,1\}^\lambda \times \{0,1\}^{\ell_{in}(\lambda)} \to \{0,1\}^{\ell_{out}(\lambda)}$, with $\lambda \in \mathbb{N}$, which appears random to an efficient quantum adversary who only sees its input/output behaviour and is ignorant of the particular key being used."




**Theorem ([Broadbent – Lord][4])**

*If the qPRF is modeled by a q. oracle, this encryption is* $\log_2(9)$*-unlconeable secure:*

$$\mathbb{P}(\mathcal{P}, \mathcal{B}, \mathcal{C} \text{ win}) \leq 9 \times 0.5^n \,.$$

*Moreover, if* $\mathcal{P}, \mathcal{B}, \mathcal{C}$ *cannot share any entanglement, then the ideal case is achieved:*

$$\mathbb{P}(\mathcal{P}, \mathcal{B}, \mathcal{C} \text{ win}) \leq 0.5^n \,.$$

[4] Broadbent and Lord. *Uncloneable Quantum Encryption via Oracles*. 2020.

- Still in the QROM model:

**Theorem ([Ananth – Kaleoglu – Li – Liu – Zhandry][5])**

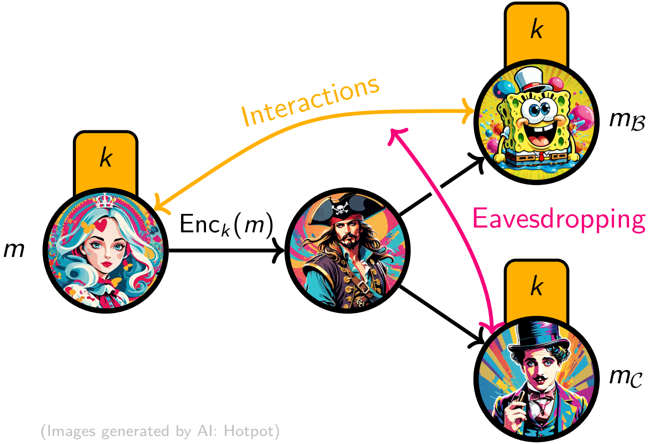
*In the QROM model, there exist encryption schemes that are uncloneable-indistinguishable secure.*

*Proof trick:* use subset coset states.

- **Remark.** When not in the QROM model, they prove that a large class of encryption schemes cannot satisfy unclonable-indistinguishability.

[5] Ananth et al. "On the Feasibility of Unclonable Encryption, and More". In: 2022.

# Attempt with Interactions and Eavesdropping Assumptions




(Images generated by AI: Hotpot)

- **Theorem ([Broadbent – Culf]):** For quantum encryption schemes of classical messages with interactive decryption, there is an equivalence between uncloneable and uncloneable-indistinguishable security.

( Broadbent and Culf. "Uncloneable Cryptographic Primitives with Interaction". In: (2023). arXiv: 2303.00048 )

- **Techniques:** Leaky MoE property and subspace coset MoE game.

# Attempts Under Other Assumptions

**Theorem ([Ananth – Kaleoglu][6])**

- *Under the assumption of post-quantum one-way functions, it is possible to turn an uncloneable encryption scheme into one with semantic security.*
- *Under the assumption of post-quantum public key encryption, it is possible to turn the scheme into a public-key uncloneable encryption scheme.*

**Theorem ([Kundu – Tan][7])**

*In a variant where $\mathcal{A}$ sends different keys to $\mathcal{B}$ and $\mathcal{C}$, the uncloneable encryption can be achieved device-independently, i.e. without trusting the quantum states and measurements used in the scheme.*

[6] Ananth and Kaleoglu. "Unclonable Encryption, Revisited". In: 2021.
[7] Kundu and Tan. *Device-independent uncloneable encryption*. 2023. arXiv: 2210.01058.

**Theorem ([Gheorghiu – Metger – Poremba][8])**

*Under the assumption of post-quantum hardness of the learning with errors (LWE) problem, there is a protocol for uncloneable encryption.*

**Theorem ([Chevalier – Hermouet – Vu][9])**

*Assume the existence of post-quantum indistinguishability obfuscation, one-way functions, and compute-and-compare obfuscation for the class of unpredictable distributions. Then:*

- *There exists a symmetric one-time unclonable encryption scheme with correctness and indistinguishable anti-piracy security;*
- *There exists a public-key reusable unclonable encryption scheme with correctness and indistinguishable anti-piracy security."*

[8] Gheorghiu, Metger, and Poremba. *Quantum cryptography with classical communication: parallel remote state preparation for copy-protection, verification, and more*. 2022. arXiv: 2201.13445.

[9] Chevalier, Hermouet, and Vu. *Unclonable Cryptography in the Plain Model*. 2023. arXiv: 2311.16663.

— Part 3 —

# Our Ideas

# 1. Half-Space Cloning

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# 3. View the Adversaries as a Cloner

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# Bibliography


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