NON LOCAL BOXES & COMMUNICATION COMPLEXITY

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Three articles: ① arXiv: 2302.00488 (PRL) with A. Broadbent, M.-O. Proulx
② arXiv: 2312.00725 (Quantum) with A. Broadbent, R. Chhaibi, I. Nechita, C. Pellegrini
③ arXiv: 2406.02199 with M. Weber.

I Motivation

⊛ Goal: Combine several principles to rule out the set of quantum correlation (Q) from the set of non-signalling correlations (NS).

⊛ Here: "Communication Complexity" (CC)

→ A collapse of CC seems "impossible" in Nature

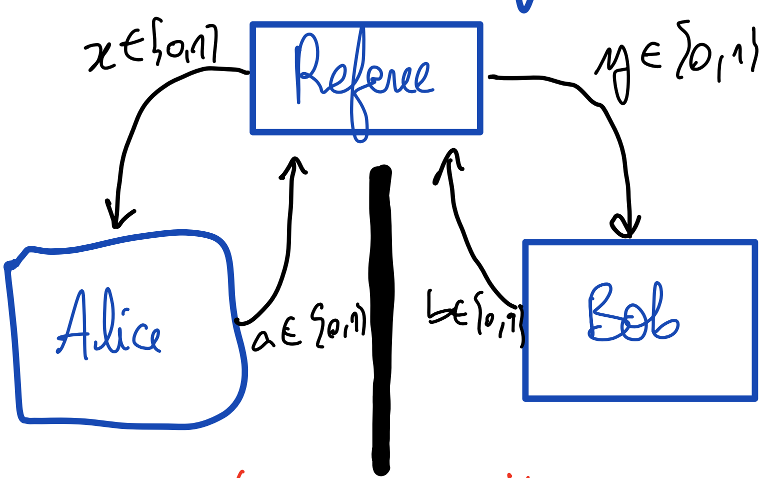
→ No quantum correlation can imply

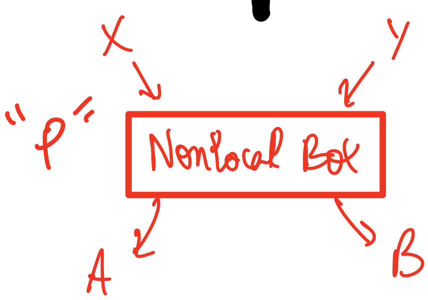
a collapse of CC, but some non-sign. imply a collapse.

(*) <u>Question</u>: What are all the non-signalling correlations that collapse CC?

II Background

1 CHSH game





s.t. ① $P(A,B|X,Y) \geq 0$

② $\sum_{A,B} P(A,B|X,Y) = 1$

③ $\sum_{A} P(A,B|X,Y) = \sum_{A} P(A,B|\hat{X},Y) =: P(B|Y)$

④ Same for $\sum_{B}$

A & B win $\Leftrightarrow$ $a \oplus b = xy$

(*) <u>Deterministic Strat.</u>

max $\mathbb{P}(\text{win}) = 75\%$

(*) <u>Classical Strat.</u>

max $\mathbb{P}(\text{win}) = 75\%$

(*) <u>Quantum Strat.</u>

max $\mathbb{P}(\text{win}) = \cos^2(\pi/8)$

$= \frac{1}{2} + \frac{1}{2\sqrt{2}} \approx 0.85\%$

(*) <u>Non-Signalling Strat.</u>

max $\mathbb{P}(\text{win}) = 100\%$

2 Examples of NLBs

① <u>Local Boxes</u>:

② <u>Quantum Box</u>

- $SR(A,B|X,Y) = \frac{1}{2}\,\delta_{A=B}$

- $I(A,B|X,Y) = \frac{1}{4}$

$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$\{E_{A|X}\}, \{F_{B|Y}\}$ POVM

$P(A,B|X,Y) = \langle\phi^+| E_{A|X} \otimes F_{B|Y} |\phi^+\rangle$

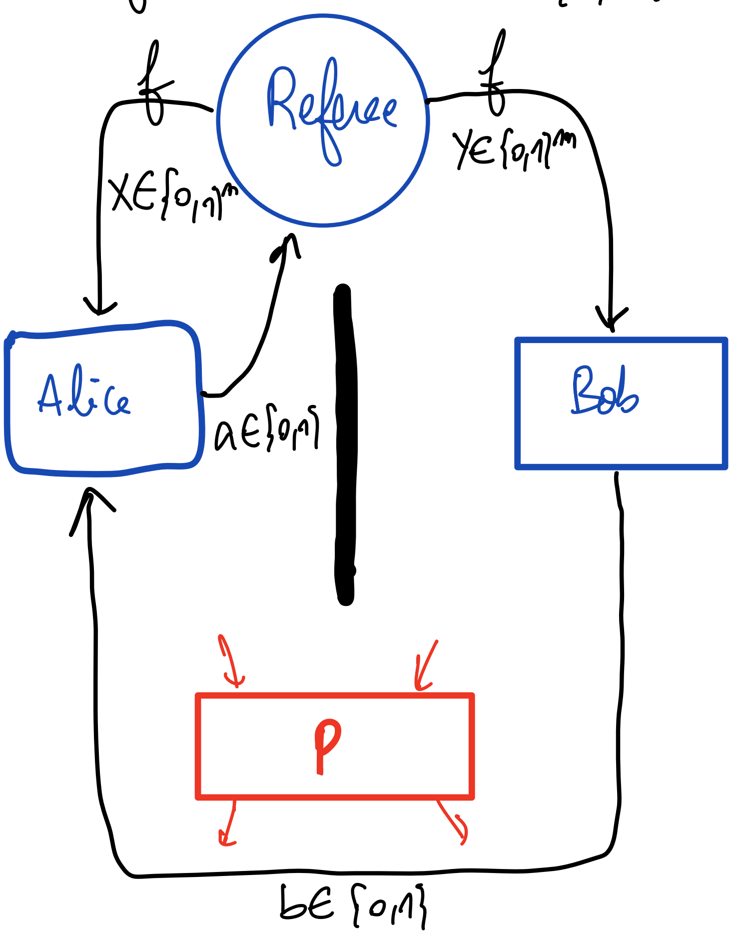
$\in Q$

③ <u>Non-Signalling Box</u>: (named after Popescu, Rohrlich)

$$PR(A,B|X,Y) = \frac{1}{2}\,\delta_{A\oplus B = XY}$$

3 Communication Complexity (CC)

$f: \{0,1\}^n \times \{0,1\}^m \to \{0,1\}$



A & B win $\Leftrightarrow$ $a = f(X,Y)$

Def: f is <u>trivial</u> in the sense of CC if A & B can perfectly win the game $\forall X, Y$.

Ex: $n = m = 2$

$X = (x_1, x_2)$, $Y = (y_1, y_2)$

① $f(X,Y) = x_1 \oplus y_1 \oplus x_2 \oplus y_2 \oplus 1$ is trivial.

② $g(X,Y) = x_1 x_2 \oplus y_1 y_2$ is trivial

③ $h(X,Y) = x_1 y_1 \oplus x_2 y_2$ is <u>NOT</u> trivial

Def: The NLB P is collapsing CC if $\exists\, p > \frac{1}{2}$ st every $f : \{0,1\}^n \times \{0,1\}^m \to \{0,1\}$ is trivial with probability $\geq p$.

Rmk: Such a collapse seems "impossible" in Nature.

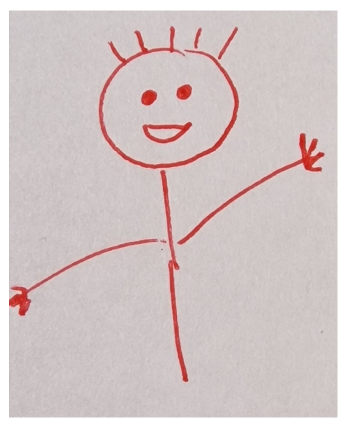
Ex:
- PR is collapsing
- $\forall P \in \mathcal{L} \cup Q$ (e.g.: SR, $\mathbb{I}$, $\langle \phi^+ | E \otimes F | \phi^+ \rangle$)

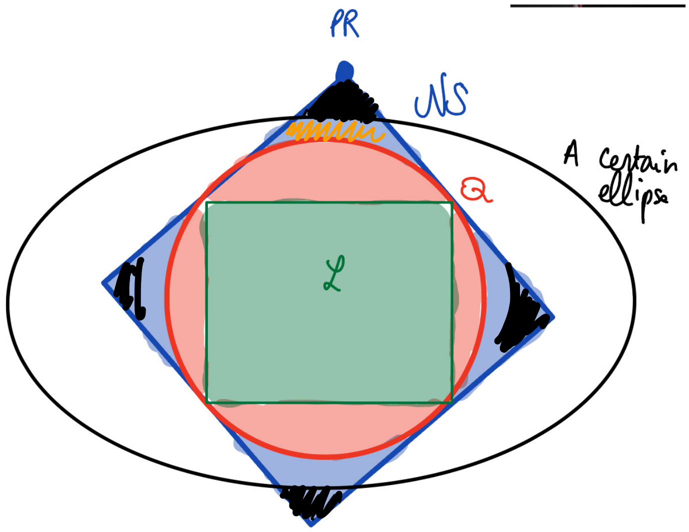
$\Rightarrow$ P does <u>not</u> collapse CC.

III Results

1 Result 1

arXiv: 2302.00488 (PRL) with A. Broadbent, M.-O. Proulx





Thm: Every NLB that is in is collapsing CC.

Sketch:

$$f : \{0,1\}^n \times \{0,1\}^m \to \{0,1\}$$

$$x \in \{0,1\}^n \,, \quad y \in \{0,1\}^m$$

① Find a protocol $P_0(f)$ that wins at guess $a, b \in \{0,1\}$ st.

$$a \oplus b = f(x,y)$$

with <u>no</u> communication, with prob. $p_0(f) > \frac{1}{2}$.

[Brassard, Buhrman, et al, PRL 96, 250401 (2006)]

② Repeat $P_0(f)$ three times

→ We get three guesses for $f(x,y)$:

$$a_1 \oplus b_1 \,, \quad a_2 \oplus b_2 \,, \quad a_3 \oplus b_3$$

→ We compute the Majority function

of the three results : it outputs the most-appearing bit in the inputs:

Ex: $(0,1,1) \mapsto 1$

$(0,0,0) \mapsto 0$

$(0,0,1) \mapsto 0$

↳ We need to use two copies of the NLB $P. \in$

→ This defines a protocol $\mathcal{P}_1(f)$ that wins at guessing $a, b \in \{0,1\}$ s.t. $a \oplus b = f(x,y)$ with prob. $p_1(f) > p_0(f)$ & with <u>no</u> communication.

③ $\forall k$, define $\mathcal{P}_{k+1}(f)$ from $\mathcal{P}_k(f)$.

s.t. $p_{k+1}(f) > p_k(f) > \dots > p_1(f) > p_0(f) > \frac{1}{2}$

with <u>no</u> communication.

④ We prove that $p_k(f) \xrightarrow[k \to +\infty]{} p_* > \frac{1}{2}$

(using a fixed-point Thm)

& prove p_* does <u>not</u> depend on f, x, y.

5 Hence: $\exists p := \frac{p_* + \frac{1}{2}}{2} > \frac{1}{2}$, $\forall f, \exists k, p_k(f) > p$

independent of f, x, y

Now, in $P_k(f)$, Alice & Bob produce $a, b \in \{0,1\}$ st $a \oplus b = f(x,y)$ with prob. $p_k(f) > p$

$\Rightarrow$ Bob can send $b \in \{0,1\}$ to Alice, & Alice knows $f(x,y)$ w.p. $p_k(f) > p$.

$\Rightarrow$ Collapse of CC. $\square$

2 Result 2

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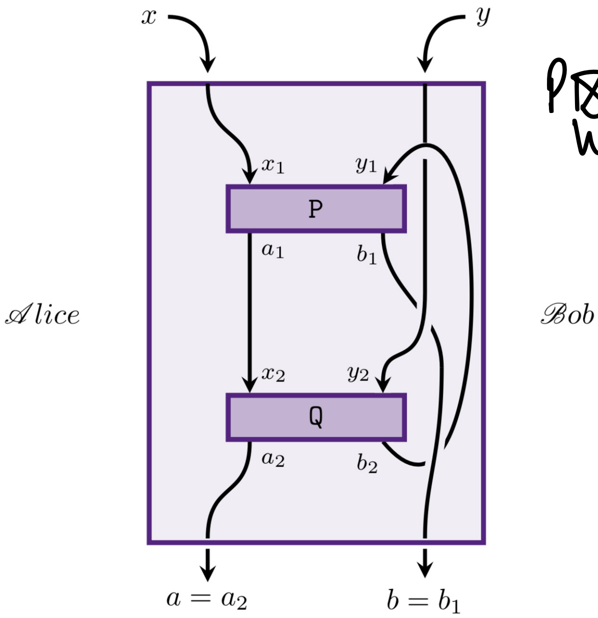
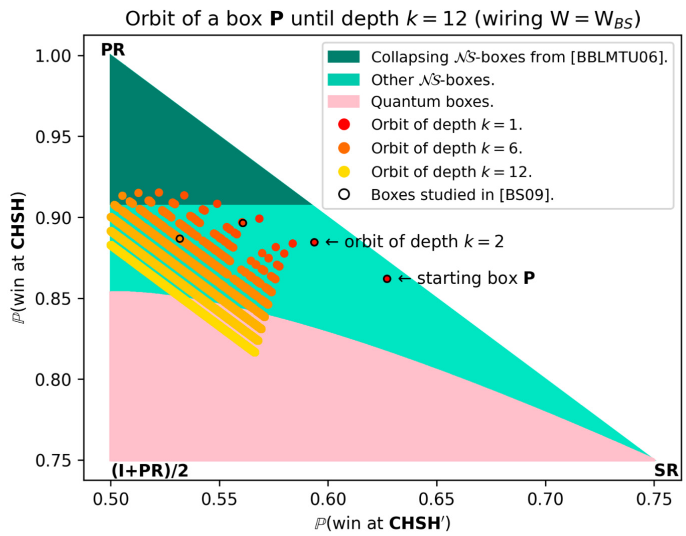
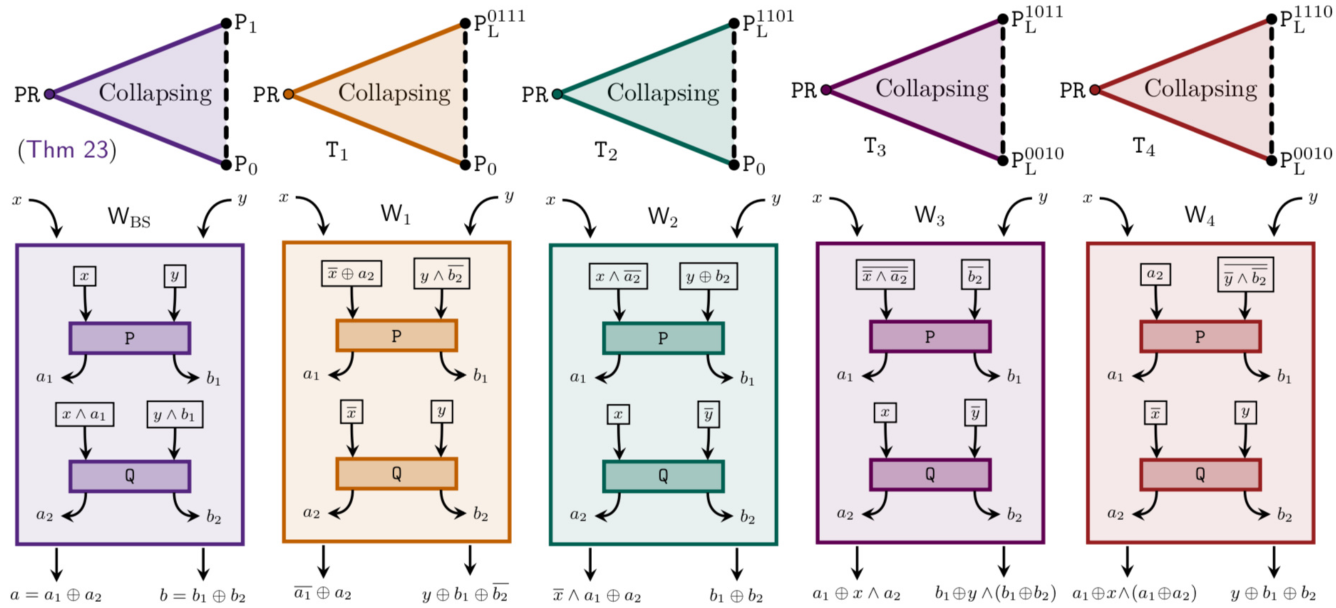
Wiring W:



$P \boxtimes_W Q$



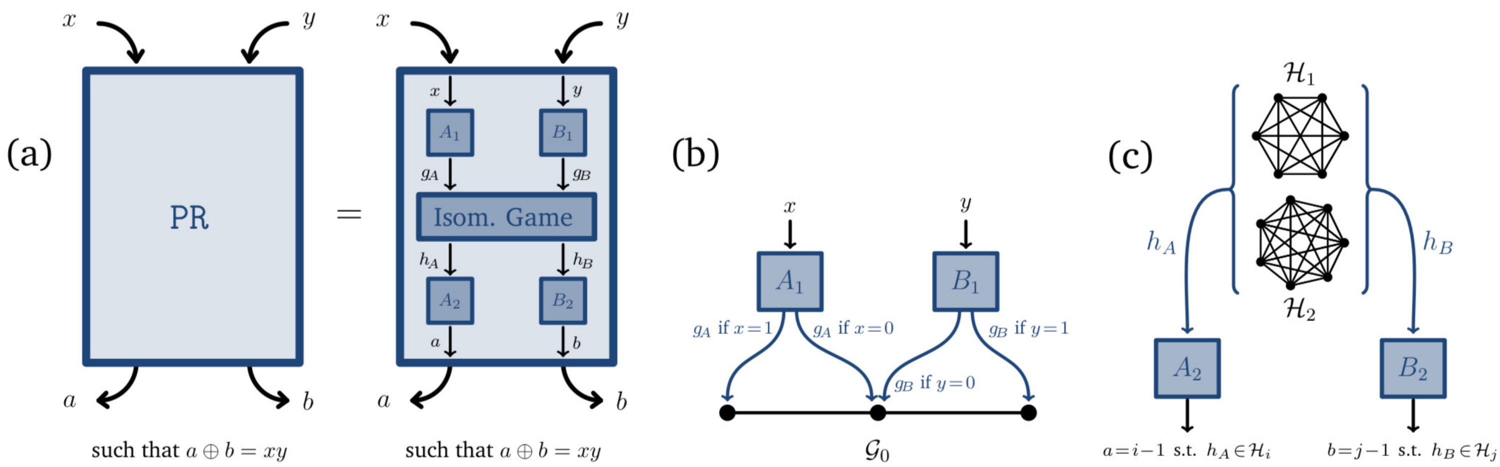


3 Result 3

arXiv: 2406.02199 with M. Weber.

CHSH game $\rightarrow$

- Graph Isom Game
- Graph Colouring Game
- Vertex Distance Game



Thank you for your attention !!