Nonlocal Games Through Communication Complexity and Quantum Cryptography

Ph.D. defense

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Manuscripts in this Thesis

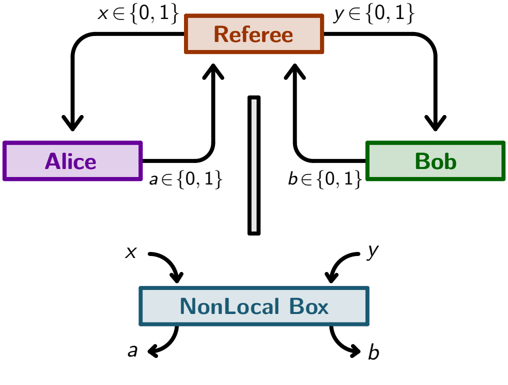
- P. Botteron, A. Broadbent, and M.-O. Proulx, "Extending the known region of nonlocal boxes that collapse communication complexity," *Physical Review Letters*, vol. 132, p. 070201, 02 (2024).
- P. Botteron, A. Broadbent, R. Chhaibi, I. Nechita, and C. Pellegrini, "Algebra of Nonlocal Boxes and the Collapse of Communication Complexity," *Quantum*, vol. 8, p. 1402, 07 (2024).
- P. Botteron and M. Weber, "Communication complexity of graph isomorphism, coloring, and distance games," arXiv:2406.02199 (2024).
- P. Botteron, A. Broadbent, E. Culf, I. Nechita, C. Pellegrini, and D. Rochette, "Towards unconditional uncloneable encryption," arXiv:2410.23064 (2024).

Contents

— Part 1 —

Background Notions

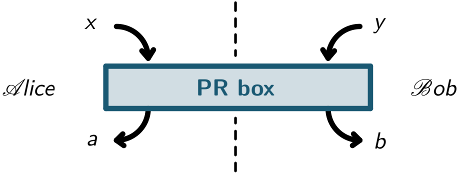
The Clauser–Horne–Shimony–Holt Game



Win at CHSH $\iff$ $a \oplus b = x\,y$.

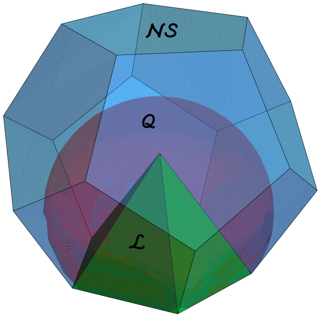
- **Strategy.** Conditional probability distribution:
$$\mathcal{S} = \Big\{ \mathrm{P} : \{0,1\}^4 \to \mathbb{R} \,:\, \mathrm{P}(a,b\,|\,x,y) \geqslant 0 \text{ and } \textstyle\sum_{a,b} \mathrm{P}(a,b\,|\,x,y) = 1 \Big\}.$$
- **Deterministic Strategies.**
$$\mathcal{L}_{\text{det}} := \Big\{ \mathrm{P} \in \mathcal{S} \,:\, \exists f, g \text{ s.t. } a = f(x) \text{ and } b = g(y) \Big\}.$$
$\rightsquigarrow \max_{\mathrm{P} \in \mathcal{L}_{\text{det}}} \mathbb{P}(\mathrm{P} \text{ win}) = 75\%.$
- **Classical Strategies.**
$$\mathcal{L} := \Big\{ \mathrm{P} = \textstyle\sum_i \lambda_i \mathrm{P}_i \,:\, \lambda_i \geqslant 0, \sum_i \lambda_i = 1, \mathrm{P}_i \in \mathcal{L}_{\text{det}} \Big\}.$$
$\rightsquigarrow \max_{\mathrm{P} \in \mathcal{L}} \mathbb{P}(\mathrm{P} \text{ win}) = 75\%.$
- **Quantum Strategies.**
$$\mathcal{Q} := \Big\{ \mathrm{P} = \langle \psi | E_{a|x} \otimes F_{b|y} | \psi \rangle \,:\, \begin{array}{c} |\psi\rangle \text{ is a quantum state} \\ \{E_{a|x}\} \,\&\, \{F_{b|y}\} \text{ are q. meas.} \end{array} \Big\}.$$
$\rightsquigarrow \max_{\mathrm{P} \in \mathcal{Q}} \mathbb{P}(\mathrm{P} \text{ win}) = \cos^2\left(\frac{\pi}{8}\right) \approx 85\%.$
- **Non-Signaling Strategies.**
$$\mathcal{NS} := \Big\{ \mathrm{P} \in \mathcal{S} : \textstyle\sum_a \mathrm{P}(a,b|x,y) = \mathrm{P}(b|y), \sum_b \mathrm{P}(a,b|x,y) = \mathrm{P}(a|x) \Big\}.$$
$\rightsquigarrow \max_{\mathrm{P} \in \mathcal{NS}} \mathbb{P}(\mathrm{P} \text{ win}) = 100\%.$

The Popescu–Rohrlich Box

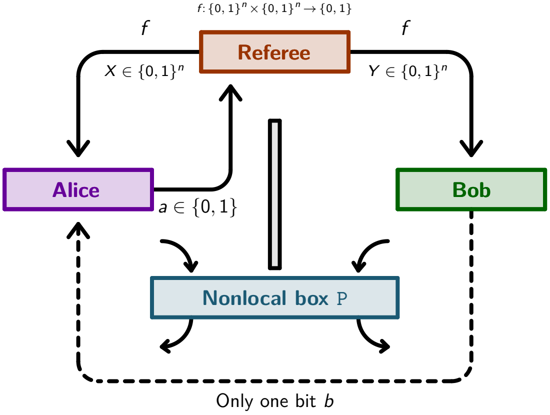


such that $a \oplus b = xy$.

$$\mathtt{PR}(a, b \mid x, y) := \frac{1}{2} \mathbf{1}_{a \oplus b = xy} .$$



Collapse of Communication Complexity



Win $\iff a = f(X,Y)$.

Def. *A function f is said to be* **trivial** *(in the sense of communication complexity) if Alice correctly guesses any value $f(X,Y)$ with only one bit transmitted from Bob to Alice, for any X and Y.*

Ex. For $n=2$, $X=(x_1,x_2)$, $Y=(y_1,y_2)$:

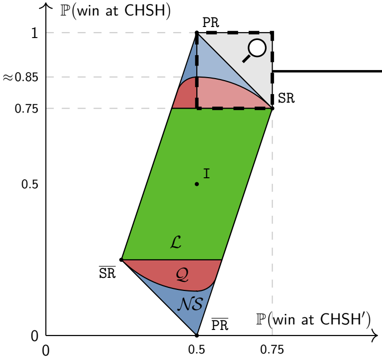
- $f := x_1 \oplus y_1 \oplus x_2 \oplus y_2 \oplus 1$ is trivial.
- $g := (x_1\, x_2) \oplus (y_1\, y_2)$ is trivial.
- $h := (x_1\, y_1) \oplus (x_2\, y_2)$ is NOT trivial.

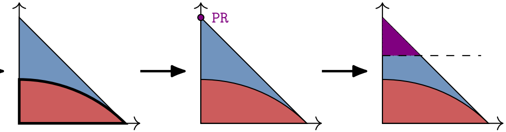
Def. *We say that a nonlocal box* P **collapses CC** *if $\exists\, \mathfrak{p} > 1/2$ such that $\forall n\in\mathbb{N}$, $\forall f : \{0,1\}^{2n}\to\{0,1\}$, and $\forall X,Y\in\{0,1\}^n$, we have:*

$$\mathbb{P}(a = f(X,Y)\,|\,X,Y,\mathrm{P}) \geqslant \mathfrak{p}\,.$$

Ex.
- The PR box is collapsing [van Dam'99].
- Local ($\mathcal{L}$) and quantum ($\mathcal{Q}$) boxes are NOT collapsing [Cleve *et al.*'99].

Collapse of Communication Complexity

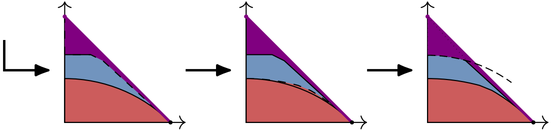




1999: Quantum boxes are **non-collapsing** [4].

1999: The PR box is **collapsing** [5].

2006: **Collapsing** region above ≈ 0.91 [6].



2009–2024: The thickened diagonal is **collapsing** (numerical results) [2, 3, 7, 8].

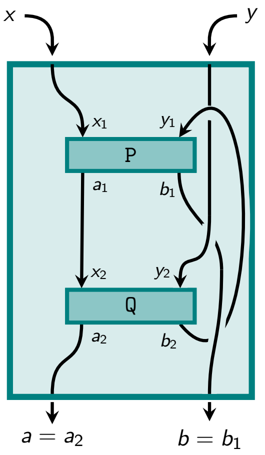
2015: Almost quantum boxes are **non-collapsing** [9].

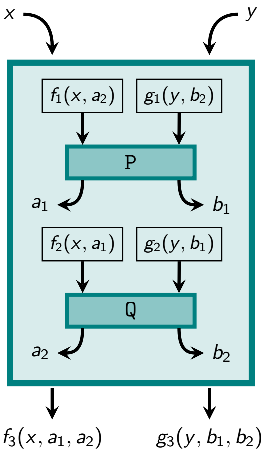
2024: **Collapsing** region above an ellipse (analytical result) [10].

— *Part* 2 —

Algebra of Boxes

Wiring of Nonlocal Boxes





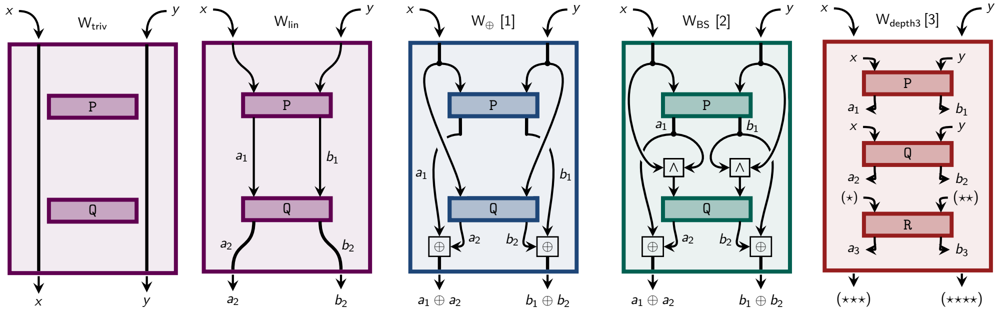
Definition. A **wiring** W between two boxes $\mathtt{P}, \mathtt{Q} \in \mathcal{NS}$ consists in six functions $f_1, f_2, g_1, g_2 : \{0,1\}^2 \to [0,1]$ and $f_3, g_3 : \{0,1\}^3 \to [0,1]$ satisfying the *non-cyclicity conditions* for all x, y:

$$f_1(x,0) \neq f_1(x,1) \Rightarrow f_2(x,0) = f_2(x,1),$$
$$f_2(x,0) \neq f_2(x,1) \Rightarrow f_1(x,0) = f_1(x,1),$$

and similarly for g_1, g_2.
The new box is denoted:

$$\mathtt{P} \boxtimes_{\mathrm{W}} \mathtt{Q} \in \mathcal{NS}\,.$$

Examples of Wirings in the Literature



where $\overline{x} := x \oplus 1$; $(\star) := xa_2 \vee x\overline{a_1} \vee \overline{x}\,\overline{a_2}a_1$; $(\star\star) := yb_2 \vee y\overline{b_1}$;

$(\star\star\star) := a_3a_2 \vee a_3\overline{a_1} \vee \overline{a_3}\,\overline{a_2}a_1$; and $(\star\star\star\star) := b_3b_2 \vee b_3\overline{b_1} \vee \overline{b_3}\,\overline{b_2}b_1$.

Algebra of Boxes

Observation: Given a wiring $\mathtt{W} = (f_1, g_1, f_2, g_2, f_3, g_3)$, the map $(\mathtt{P}, \mathtt{Q}) \mapsto \mathtt{P} \boxtimes_{\mathtt{W}} \mathtt{Q}$ is bilinear:

$$\mathtt{P} \boxtimes_{\mathtt{W}} \mathtt{Q}\big(a, b \,|\, x, y\big) := \sum_{a_1, a_2, b_1, b_2} \mathtt{P}\big(a_1,\ b_1 \,|\, f_1(x, a_2),\ g_1(y, b_2)\big) \\ \times \mathtt{Q}\big(a_2,\ b_2 \,|\, f_2(x, a_1),\ g_2(y, b_1)\big) \times \mathbf{1}_{a=f_3(x,a_1,a_2)} \times \mathbf{1}_{b=g_3(y,b_1,b_2)}\,.$$

$\Rightarrow$ The vector space $\mathcal{B}_{\mathtt{W}} := \big(\{boxes\}, \boxtimes_{\mathtt{W}}\big)$ is an algebra, that we call the **algebra of boxes**.

Proposition (B.–Broadbent–Chhaibi–Nechita–Pellegrini'24)

Assume W is a wiring such that $f_1 = f_2 = f(x)$ and $g_1 = g_2 = g(y)$. Then:

1. $\mathcal{B}_{\mathtt{W}}$ is commutative $\iff$ $f_3(x, a_1, a_2) = f_3(x, a_2, a_1)$ and $g_3(y, b_1, b_2) = g_3(y, b_2, b_1)$.

If in addition $f(x) = x$ and $g(y) = y$:

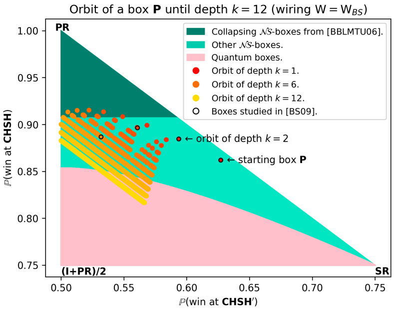
2. $\mathcal{B}_{\mathtt{W}}$ is associative $\iff$ $f_3(x, a_1, f_3(x, a_2, a_3)) = f_3(x, f_3(x, a_1, a_2), a_3)$ and $g_3(y, b_1, g_3(y, b_2, b_3)) = g_3(y, g_3(y, b_1, b_2), b_3)$.

Orbit of a Box

$$\mathrm{Orbit}_{\mathrm{W}}^{(3)}(\mathrm{P}) = \left\{ (\mathrm{P} \boxtimes \mathrm{P}) \boxtimes \mathrm{P},\ \mathrm{P} \boxtimes (\mathrm{P} \boxtimes \mathrm{P}) \right\},$$

$$\mathrm{Orbit}_{\mathrm{W}}^{(4)}(\mathrm{P}) = \Big\{ \big((\mathrm{P}\boxtimes\mathrm{P})\boxtimes\mathrm{P}\big)\boxtimes\mathrm{P},\ \big(\mathrm{P}\boxtimes(\mathrm{P}\boxtimes\mathrm{P})\big)\boxtimes\mathrm{P},\ (\mathrm{P}\boxtimes\mathrm{P})\boxtimes(\mathrm{P}\boxtimes\mathrm{P}),\ \mathrm{P}\boxtimes\big((\mathrm{P}\boxtimes\mathrm{P})\boxtimes\mathrm{P}\big),\ \mathrm{P}\boxtimes\big(\mathrm{P}\boxtimes(\mathrm{P}\boxtimes\mathrm{P})\big) \Big\},$$

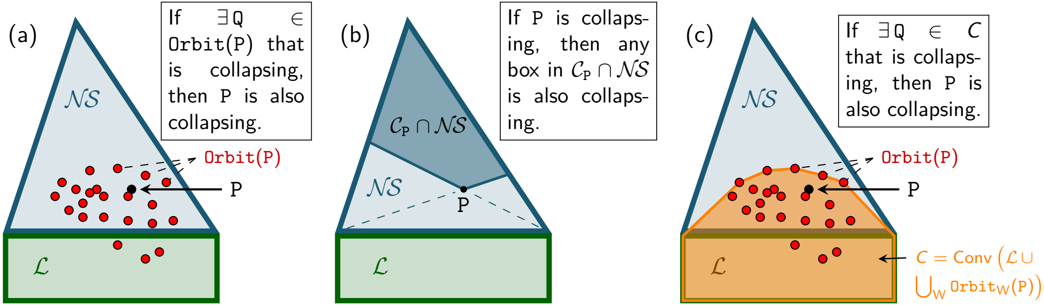
$\mathrm{Orbit}_{\mathrm{W}}^{(k)}(\mathrm{P}) := \big\{$ all possible products with k times the term P, using the multiplication $\boxtimes_{\mathrm{W}}$ $\big\}$.



Theorem (B.–Broadbent–Chhaibi–Nechita–Pellegrini'24)

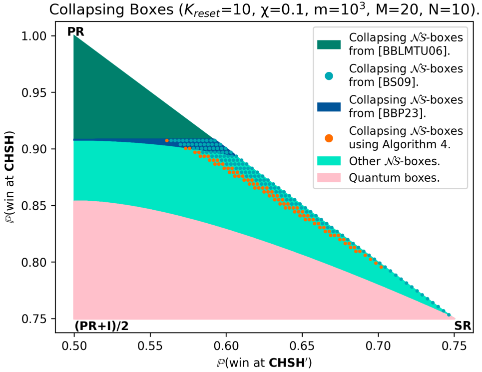
For fixed k, the points of the k-orbit are aligned, and the highest CHSH-value is achieved by the parenthesization with multiplication only on the right: $\mathrm{P}^{\boxtimes k} := \Big(\big((\mathrm{P} \boxtimes \mathrm{P}) \boxtimes \mathrm{P}\big) \cdots \Big) \boxtimes \mathrm{P}\,.$

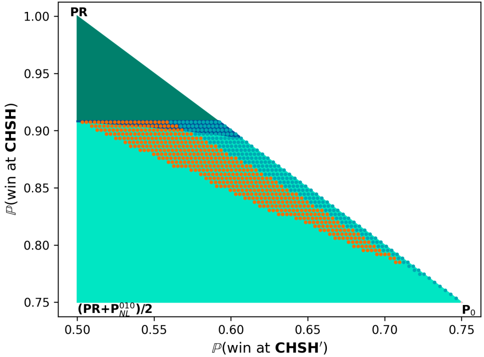
Here is the consequence to Communication Complexity:



Numerical Results

Using a gradient descent algorithm, we obtain in **orange** new collapsing boxes (this result is similar to the independent and concurrent work of [Eftaxias et al.'23] [3]):





Collapse of CC from Multiplication Tables

Theorem (B.–Broadbent–Chhaibi–Nechita–Pellegrini'24)

Let $\mathtt{Q}, \mathtt{R} \in \mathcal{NS}$ be boxes. Assume there exists a wiring $\mathsf{W} \in \mathcal{W}$ that induces the following multiplication table:

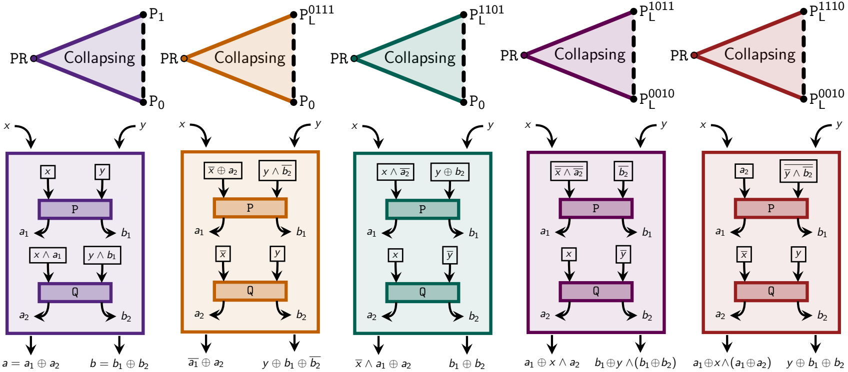
	$\mathtt{PR}$	$\mathtt{Q}$	$\mathtt{R}$
$\mathtt{PR}$	$\mathtt{PR}$	$\mathtt{PR}$	$\mathtt{PR}$
$\mathtt{Q}$	$\frac{1}{2}\left(\mathtt{Q}+\mathtt{R}\right)$	$\mathtt{Q}$	$\mathtt{R}$
$\mathtt{R}$	$\mathtt{PR}$	$\mathtt{R}$	$\mathtt{Q}$

Then the triangle $\mathrm{Conv}\{\mathtt{PR}, \mathtt{Q}, \mathtt{R}\} \setminus \mathrm{Conv}\{\mathtt{Q}, \mathtt{R}\}$ is collapsing.

Collapse of CC

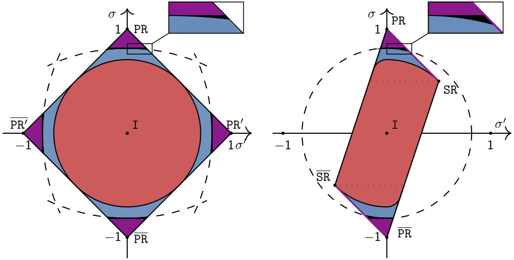
Corollary (B.–Broadbent–Chhaibi–Nechita–Pellegrini'24)

All the triangles drawn below are collapsing.

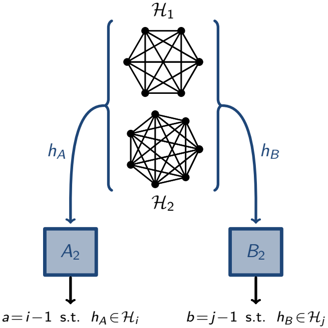


Examples of Other Methods to Collapse CC

B.–Broadbent–Proulx, PRL:132 (7 2024) [10].
Using bias amplification by majority function, one can prove that all the boxes above an explicit ellipse collapse CC:



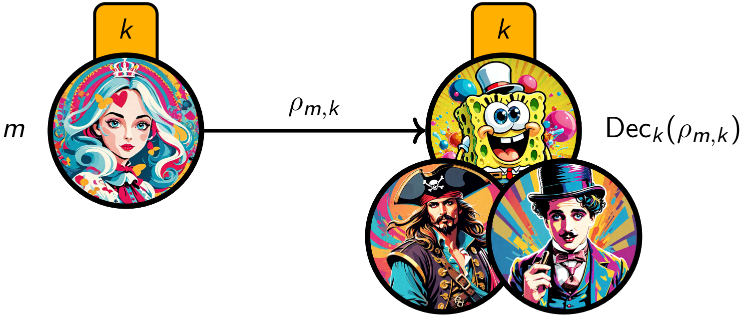
B.–Weber, arXiv:2406.02199 [11].
In other nonlocal games related to graphs, one can show that some non-signaling correlations collapse CC:



— *Part 3* —

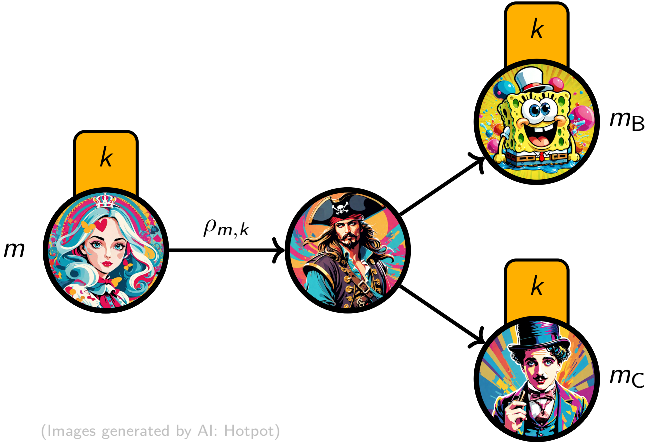
Unclonable Bit

Scenario



Correctness: $\forall m, \forall k, \quad \mathrm{Dec}_k(\rho_{m,k}) \overset{a.s.}{=} m.$

No-Cloning Game



(Images generated by AI: Hotpot)

- **Rule:** The malicious team $(\mathrm{P}, \mathrm{B}, \mathrm{C})$ wins iff. $m_{\mathrm{B}} = m_{\mathrm{C}} = m$.
- **Def (Unclonable-Indistinguishable Security):** The encryption scheme $(m, k) \mapsto \rho_{m,k}$ is said *weakly secure* if:

$$\mathbb{P}\Big((\mathrm{P}, \mathrm{B}, \mathrm{C}) \text{ win}\Big) \leqslant \frac{1}{2} + f(\lambda),$$

where $\lim f(\lambda) = 0$, and where λ is the security parameter. It is *strongly secure* if $f(\lambda) = \mathrm{negl}(\lambda)$.
- **Unclonable Bit Problem (**Broadbent–Lord'20**):** Is there an encryption scheme $(m, k) \mapsto \rho_{m,k}$ that is both correct and strongly secure?

Mathematical Translation

The winning probability at the no-cloning game is expressed as follows:

$$\mathbb{P}\Big((\mathsf{P},\mathsf{B},\mathsf{C}) \text{ win} \ \Big| \ \rho_{m,k}\Big) = \mathop{\mathbb{E}}_{\substack{m\in\{0,1\} \\ k\leftarrow \mathsf{Gen}(1^\lambda)}} \sum_{m_\mathsf{B},m_\mathsf{C}\in\{0,1\}} \mathbf{1}_{\{m_\mathsf{B}=m_\mathsf{C}=m\}} \operatorname{Tr}\Big[\Phi(\rho_{m,k})\big(B_{m_\mathsf{B}|k}\otimes C_{m_\mathsf{C}|k}\big)\Big]$$

$$= \mathop{\mathbb{E}}_{m,k} \operatorname{Tr}\Big[\Phi(\rho_{m,k})\big(B_{m|k}\otimes C_{m|k}\big)\Big].$$

Goal

Find the most secure encryption scheme against the strongest attack, *i.e.* solve:

$$\inf_{\rho_{m,k}} \sup_{\Phi,\{B_{i|k}\},\{C_{j|k}\}} \mathop{\mathbb{E}}_{m,k} \operatorname{Tr}\Big[\Phi(\rho_{m,k})\big(B_{m|k}\otimes C_{m|k}\big)\Big].$$

Candidate Scheme

Let $k \in \{1, .., K\}$. Consider a family $\{\Gamma_1, \ldots, \Gamma_K\}$ that is:

- Hermitian (*i.e.* $\Gamma_k^\dagger = \Gamma_k$ for all k); and
- Unitary (*i.e.* $\Gamma_K^\dagger \Gamma_k = \Gamma_K \Gamma_k^\dagger = \mathbb{I}$ for all k); and
- Pairwise anti-commuting (*i.e.* $\Gamma_k \Gamma_j = -\Gamma_j \Gamma_k$ for all $j \neq k$).

Why? Because then $\left\| \sum_{k=1}^K v_k \Gamma_k \right\|_{\mathrm{op}} = \|v\|_2$ for any $v = (v_1, .., v_K) \in \mathbb{R}^K$, and in particular:

$$\left\| \sum_{k=1}^K \Gamma_k \right\|_{\mathrm{op}} = \left\| (1, .., 1) \right\|_2 = \sqrt{1^2 + \cdots + 1^2} = \sqrt{K}.$$

Candidate Scheme

For $m \in \{0, 1\}$ and $k \in \{1, .., K\}$, consider:

$$\rho_{m,k} := \frac{2}{d} \, \frac{\mathbb{I}_d + (-1)^m \Gamma_k}{2}.$$

Observation

This scheme is correct.

Proof. Given k and $\rho_{m,k}$, measure $\rho_{m,k}$ in an eigenbasis of Γ_k. Obtain 1 or -1, and recover the value of m. □

Upper Bound on the Winning Probability

Using the candidate scheme, we obtain the following upper bound on the *best* winning probability (where the U_k are Hermitian unitaries):

$$\mathbb{P}^*\Big((\mathrm{P},\mathrm{B},\mathrm{C})\text{ win}\Big) \leqslant \frac{1}{4} + \frac{1}{4K} \sup_{\{U_k\}} \underbrace{\left\| \sum_{k=1}^{K} \Big(\Gamma_k \otimes U_k \otimes \mathbb{I}_D + \Gamma_k \otimes \mathbb{I}_D \otimes U_k + \mathbb{I}_d \otimes U_k \otimes U_k \Big) \right\|_{\mathrm{op}}}_{=:W_K(U_1,..,U_K)} .$$

Theorem
(B.–Broadbent–Culf–Nechita–Pellegrini–Rochette'24)

If for all Hermitian unitaries $U_1, .., U_K$:

$$\Big\| W_K(U_1, .., U_K) \Big\|_{\mathrm{op}} \leqslant K + 2\sqrt{K} ,$$

then, the scheme defined by the Γ_k's is weakly secure:

$$\mathbb{P}\Big((\mathrm{P},\mathrm{B},\mathrm{C})\text{ win the game}\Big) \leqslant \frac{1}{2} + \frac{1}{2\sqrt{K}} .$$

Now, we want to prove:

Conjecture

Let $K \geqslant 2$ be an integer, $\Gamma_1, .., \Gamma_K$ be Hermitian unitaries that pairwise anti-commute, and $U_1, .., U_K$ be Hermitian unitaries. Then:

$$\sup_{\{\Gamma_k\},\{U_k\}} \left\| \sum_{k=1}^{K} \Big(\Gamma_k \otimes U_k \otimes \mathbb{I} + \Gamma_k \otimes \mathbb{I} \otimes U_k + \mathbb{I} \otimes U_k \otimes U_k \Big) \right\|_{\mathrm{op}} \leqslant K + 2\sqrt{K}\,.$$

Observation 1

The value $K + 2\sqrt{K}$ is achieved when considering $U_k = \mathbb{I}$ for all k.

Proof. $\left\| \sum_k (2\,\Gamma_k + \mathbb{I}) \right\|_{\mathrm{op}} = \left\| 2\,(\sum_k \Gamma_k) + K\,\mathbb{I} \right\|_{\mathrm{op}} = 2\,\left\| \sum_k \Gamma_k \right\|_{\mathrm{op}} + K = 2\sqrt{K} + K\,.$ □

True in the Commuting Case

Observation 2

The Conjecture holds if we assume that the operators U_k commute.

Proof. If the operators U_k commute, then they are diagonalizable in a common basis. But they are Hermitian and unitaries, so their eigenvalues are ± 1 and we may assume:

$$U_k = \begin{pmatrix} \pm 1 & & \\ & \ddots & \\ & & \pm 1 \end{pmatrix}.$$

Then, using the triangular inequality, we obtain:

$$\begin{aligned} \left\| W_K \right\|_{\mathrm{op}} &\leqslant \left\| \sum_{k=1}^{K} \Gamma_k \otimes (\pm 1) \otimes 1 \right\|_{\mathrm{op}} + \left\| \sum_{k=1}^{K} \Gamma_k \otimes 1 \otimes (\pm 1) \right\|_{\mathrm{op}} + \sum_{k=1}^{K} \left\| \begin{pmatrix} \pm 1 & & \\ & \ddots & \\ & & \pm 1 \end{pmatrix} \right\|_{\mathrm{op}} \\ &= \left\| \textstyle\sum_{k=1}^{K} \Gamma_k \right\|_{\mathrm{op}} + \left\| \textstyle\sum_{k=1}^{K} \Gamma_k \right\|_{\mathrm{op}} + \textstyle\sum_{k=1}^{K} 1 = \sqrt{K} + \sqrt{K} + K\,. \quad \square \end{aligned}$$

Conjecture in the General Case

Recall: $W_K(U_1, .., U_K) := \sum_{k=1}^{K} \Big(\Gamma_k \otimes U_k \otimes \mathbb{I} + \Gamma_k \otimes \mathbb{I} \otimes U_k + \mathbb{I} \otimes U_k \otimes U_k \Big).$

Conjecture: $\forall U_1, ..., U_K, \quad \Big\| W_K(U_1, .., U_K) \Big\|_{\mathrm{op}} \leqslant K + 2\sqrt{K}.$

Theorem (B.–Broadbent–Culf–Nechita–Pellegrini–Rochette'24)

The Conjecture is valid for small key sizes ($K \leqslant 7$).

Proof Idea. When $K \leqslant 7$, we find an explicit sum-of-squares (SoS) decomposition:

$$\left(K + 2\sqrt{K}\right) \mathbb{I} - W_K = \sum_{k=1}^{K} \alpha_k A_k^2$$

for some explicit coefficients $\alpha_k \geqslant 0$ and operators A_k. Hence $\left(K+2\sqrt{K}\right) \mathbb{I} - W_K \succcurlyeq 0$ and $K+2\sqrt{K} \geqslant \|W_K\|_{\mathrm{op}}$. □

Numerical Evidence for Larger Key Sizes

The Conjecture is also numerically confirmed:

- at least for $K \leqslant 17$ with the NPA level-2 algorithm, and
- at least for $K \leqslant 18$ using the Seesaw algorithm.

The complete proof (for all $K \in \mathbb{N}$) is open.

Conclusion

Conclusion

Summary

- We prove the collapse of communication using various methods: wiring of boxes, bias amplification, and graph properties.
- We propose a candidate scheme for the unclonable bit problem in the plain model. We partially prove the weak security and provide numerical evidence that it holds for any key size.

Future Work

- Find other methods to discard non-physical correlations using communication complexity or any other information-based principle.
- Study the strong security in the unclonable bit problem.

Thank you!

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